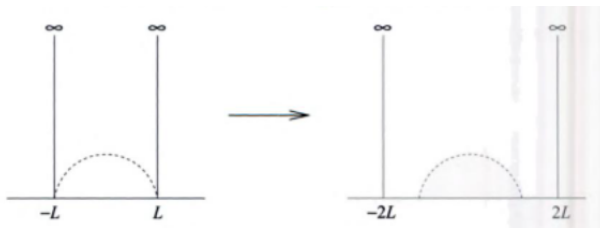
Quantum Mechanics
ISI B.Math/M.Math
Backpaper Exam : June 4,2025

Total Marks: 75
Time : 3 hours
Answer all questions

1.(**Marks = 5 + 5 +5**)



A particle of mass m is in an infinite one-dimensional box with walls at $x = -L$ and $x = L$ and is in its ground state $\psi_0(x)$ at $t = 0$. Assume now that at $t = 0$ the walls of the box move instantaneously so that its width doubles ($-2L < x < 2L$). This change does not affect the state of the particle which remains the same before and after (i.e, $\psi_0(x)$ of the box of width $2L$.)

(a) Write down the wave function of the particle at time $t > 0$.

(b) Calculate the probability P_n of finding the particle in an arbitrary stationary state $\tilde{\psi}_n(x)$ of the modified system. What is the probability of finding the system in an odd state ?

(c) What is $< H >$ = the expectation value of the energy at time $t > 0$?

[You can use : $\sum_{n=0}^{\infty} \frac{(2n+1)^2}{[(2n+1)^2-4]^2} = \frac{\pi^2}{16}$]

2.(**Marks = 8 + 7**)

(a) A one dimensional harmonic oscillator of mass m has potential energy $V(x) = \frac{1}{2}m\omega^2x^2$. Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$
Find the uncertainty product $\Delta x\Delta p$ in the nth eigenstate belonging to the eigenvalue $E_n = (n+\frac{1}{2})\hbar\omega$ where $n = 0, 1, 2, \cdots$ and show that the ground state of the harmonic oscillator is the minimum uncertainty state.

(b) Let us now consider a three dimensional isotropic harmonic oscillator of mass m with a potential given by $V(\mathbf{r}) = \frac{1}{2}m\omega^2r^2$. Find the energy eigenvalues for such an oscillator. What is the degree of degeneracy of the eigenstates ?

3.(**Marks = 2 + 6 + 6 + 6**)

An electron is in the spin state

$$\chi = A\begin{pmatrix}3i\\4\end{pmatrix}$$

(a) Determine the normalization constant A

(b) Find the expectation value of S_x, S_y and S_z

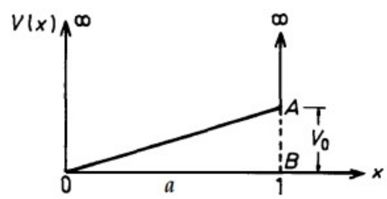
(c) Find the uncertainties $\sigma_{S_x}, \sigma_{S_y}, \sigma_{S_z}$. [Note: These σs are the uncertainties, not the Pauli matrices]

(d) Confirm that your results are consistent with all three uncertainty principles.

4. (**Marks = 10 + 5**)

(a) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ where L_z is the z component of the angular momentum and $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

(b) Show that the Hamiltonian $H = \frac{p^2}{2m} + V(r)$ commutes with all three components of angular momentum as long as the potential depends only on r.



5. (**Marks = 10**)

Employing first order perturbation theory , calculate the energies of the first three states of an infinite square well of width a, whose portion AB has been sliced off (see figure). Note that OA is a straight line.

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information you may (or may not) need :

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$